Commodity Taxes

Wojciech Kopczuk, adapted by Kyle Coombs

Vassar College

September 30, 2025

A new Congressional Request

- Imagine Kirsten Gillibrand and Pat Ryan call again:

  Remember how we ended global warming, all war, and busted all inefficient monopolies? Well we decided not to tax just one good cause that'll hurt people too much. Instead, we'll tax everything a little. Should the tax rate be uniform or vary?

- Woof. Is this a better idea than taxing just one good?
- Do you think anything should still be tax-exempt?

Today's lecture will help you answer that question.

Lecture goals

- Define a simplified optimization problem of the government
- Understand how the optimal tax accounts for individual utility maximization
- Derive and interpret the inverse elasticity rule of optimal commodity taxation

Optimal taxation

The problem: the government must collect revenue R, how can it structure taxation to maximize welfare (minimize excess burden)?

What is the optimal way to raise revenue? (Hint: recall the 2nd welfare theorem) **Answer:** differentiated lump-sum taxes

The logic: (1) lump-sum tax minimizes behavioral costs and (2) taxes may be differentiated by ability to pay.

Problem: Ability pay is unobservable, so you can't really implement lump-sum taxes in practice

Optimal commodity taxation

In 1927, Arthur Pigou asked Frank Ramsey:

- If some goods untaxable and varied lump sum taxes impossible
- And many ways to raise the same revenue R
- How should we tax K goods to raise a specific R while minimizing the impact on welfare?
- Is uniform taxation: $t_1 = t_2 = \cdots = t_K$ best?

Ramsey found uniform taxes are not optimal and proposed:

$$\textbf{Marginal Revenue}_i \times \lambda = \textbf{Marginal Excess Burden}_i$$

λ: marginal value of revenue (how helpful is the revenue?)

Intuition: goods with lower EB per dollar of revenue, taxed more

$$EB = \frac{1}{2} \frac{\varepsilon_S \varepsilon_D}{\varepsilon_S - \varepsilon_D} \frac{Q}{p} (\Delta t)^2$$

Logical chain: Higher the elasticity of good $\Rightarrow$ higher the excess burden of taxing it $\Rightarrow$ the lower the optimal tax on that good

Assumptions

Assume additively separable and quasilinear utility with 3 goods:

$$u_1(C_1) + u_2(C_2) + C_3$$

- additively separable: marginal utilities independent, so MRS of each pair of goods is independent of the third
- quasilinear means utility is linear in a numéraire good, C_3

the representative consumer's budget constraint is

$$(p_1 + t_1)C_1 + (p_2 + t_2)C_2 + C_3 = y$$

and the government's revenue constraint is

$$t_1 C_1 + t_2 C_2 = R$$

Firms are competitive with perfectly elasticity supply curves

Comprehension checks

- Is $U = ln(x) + ln(y) + z$ quasilinear and additively separable?
- Do changes in x affect y or z?
  - Additive separability removes substitution/complementarity
- Which has highest marg. utility for $\{x, y, z\} > 1$?
  - z! (the numéraire) by quasilinearity
  - all new income spent on z, no income effects on x or y
- How do these assumptions simplify the problem?
  - Much simpler marginal utility functions
  - Isolates price effect of taxes by zeroing out income effects
- Imagine the government wanted to raise \$100 in revenue by taxing $C_1 = 10$ and $C_2 = 50$. What are possible tax rates?

Government's problem

- Select t_1 and t_2 in order to... maximize utility

$$u_1(C_1) + u_2(C_2) + C_3$$

- Subject to two constraints:
  - Individual optimization: C_1, C_2 and C_3 are selected to maximize utility given the choice of t_1 and t_2
  - Revenue constraint: $t_1 C_1 + t_2 C_2 = R$

Individual problem

- Individual optimization:

$$\max_{C_1, C_2} u_1(C_1) + u_2(C_2) + y - (p_1 + t_1)C_1 - (p_2 + t_2)C_2$$

- ... gives solutions for consumption of goods $i = 1, 2$: $C_i^*(p_1 + t_1, p_2 + t_2, y)$ that satisfy:

$$MU_1 = u_1'(C_1^*) = p_1 + t_1 \quad \text{and} \quad MU_2 = u_2'(C_2^*) = p_2 + t_2$$

  so that we can just write $C_1^*(p_1 + t_1)$ and $C_2^*(p_2 + t_2)$.[1]

[1] I will write C_1^* and C_2^* without arguments rather than $C_1^*(p_1 + t_1)$ or $C_2^*(p_2 + t_2)$ to keep notation manageable but remember that these are solutions (endogenous variables) that depend on the prices/taxes in place

Envelope theorem

- An important observation: the derivative of utilty with respect to t_i is simple at the optimum (written here for ${t_1}^2$):

$$\frac{\partial}{\partial C_1}\left\{u_1(C_1^*)+u_2(C_2^*)+y-(p_1+t_1)C_1^*-(p_2+t_2)C_2^*\right\}\cdot\frac{\partial C_1}{\partial t_1}-C_1^*$$

- How is that simple? The first term is zero because utility is maximized (first-order condition for optimization)!
- The effect of a change in t_1 on the utility level is simply $-C_1^*$, and the effect of a change in t_2 on the utility level is $-C_2^*$.
  - If tax increases, you're out what you paid in taxes: $t_i \times C_i$
- An example of the much more general result known as "the envelope theorem"

[2] note chain rule + product rule

Commodity taxation (continued)

- Note $C_3 = y - (p_1 + t_1) \times C_1^* - (p_2 + t_2) \times C_2^*$, so the government maximizes:

$$u_1(C_1^*) + u_2(C_2^*) + \underbrace{y - (p_1 + t_1) \cdot C_1^* - (p_2 + t_2) \cdot C_2^*}_{\text{this is } C_3}$$

  subject to

$$t_1 C_1^* + t_2 C_2^* = R$$

- We will characterize the solution as follows. Suppose that the government maximizes a weighted sum of utility and revenue

$$\max_{t_1, t_2} u_1(C_1^*) + u_2(C_2^*) + y - (p_1 + t_1) C_1^* - (p_2 + t_2) C_2^* + \lambda \cdot (t_1 C_1^* + t_2 C_2^*)$$

  where λ is the weight assigned to revenue relative to utility

  Note: R is intentionally left out, you'll see why soon

Maximization using Lagrange Multiplier Approach

$$\max_{t_1,t_2} u_1(C_1^*)+u_2(C_2^*)+y-(p_1+t_1)C_1^*-(p_2+t_2)C_2^*+\lambda\cdot(t_1C_1^*+t_2C_2^*)$$

Why does this yields tax levels that max welfare subject to revenue constraint?

- Solving for a given value of λ yields $R = t_1C_1^* + t_2C_2^*$
- For each pair (R, λ), utility maximized. If not, other values of t_1 and t_2 would yield
  - the same revenue $t_1C_1^* + t_2C_2^*$
  - more welfare $u_2(C_1^*) + u_2(C_2^*) + y - (p_1 + t_1)C_1^* - (p_2 + t_2)C_2^*$
- ...but that would mean we did not maximize the objective!
- Different values of λ correspond to different revenue levels.
- If you pick an R, there is a corresponding λ

First-order condition

$$\max_{t_1, t_2} u_1(C_1^*) + u_2(C_2^*) + y - (p_1 + t_1)C_1^* - (p_2 + t_2)C_2^* + \lambda \cdot (t_1 C_1^* + t_2 C_2^*)$$

- How do we maximize this monstrosity?
- Take 1st-order conditions[3] with respect to t_1 and t_2.
- For t_1, recall that the effect on utility is equal to $-C_1^*$
- The total effect on government objective is:

$$-C_1^* + \lambda \cdot \left(C_1^* + t_1 \frac{\partial C_1^*}{\partial t_1} \right) = 0$$

- Note:
  - The effect on the objective should be zero at the optimum
  - C_2^* does not depend on t_1 — major simplification due to the structure of utility function that we assumed.

[3] fancy way of saying the derivative

Marginal cost of funds

- Thus:

$$\lambda = \frac{C_1^*}{C_1^* + t_1 \frac{\partial C_1^*}{\partial t_1}} \text{ and } \lambda = \frac{C_2^*}{C_2^* + t_2 \frac{\partial C_2^*}{\partial t_2}}$$

- Called the "Marginal Cost of Funds" (MCF) for tax instruments, the loss of welfare per dollar of revenue

$$\text{MCF} = \frac{\text{effect of the tax instrument on welfare}}{\text{effect of the tax instrument on revenue}} = \frac{MEB}{MR}$$

- If positive, the tax instrument raised welfare (e.g. fixed market failure) and may be called the "Marginal Value of Public Funds" (MVPF)

Optimum

- In general: the MCF of all taxes should be equal. Why?
- If MCF for good 1 exceeds that of good 2, you can reduce t_1 and increase t_2 to keep revenue fixed and increase welfare
- Equalizing means:

$$\frac{C_1^*}{C_1^* + t_1 \frac{\partial C_1^*}{\partial t_1}} = \lambda = \frac{C_2^*}{C_2^* + t_2 \frac{\partial C_2^*}{\partial t_2}}$$

- If we set all equal, we can drop λ and back out tax rates
- Everything that follows works regardless of the value of λ and the corresponding revenue $R = t_1 C_1^* + t_2 C_2^*$

Optimum — the inverse elasticity

- Let's re-arrange our optimality condition

$$\frac{1}{1 + t_1 \frac{\partial C_1^*}{\partial t_1} \Big/ C_1^*} = \frac{1}{1 + t_2 \frac{\partial C_2^*}{\partial t_2} \Big/ C_2^*} \Rightarrow t_1 \frac{\partial C_1^*}{\partial t_1} \Big/ C_1^* = t_2 \frac{\partial C_2^*}{\partial t_2} \Big/ C_2^*$$

- Define elasticity of demand to the price:

$$\eta_i = \frac{\partial C_i^*}{\partial t_i} \frac{p_i + t_i}{C_i^*} \approx \frac{\partial C_i^*}{\partial t_i} \frac{p_i}{C_i^*}$$

- The optimal commodity taxes should satisfy

$$\frac{t_1}{p_1 + t_1} \eta_1 = \frac{t_2}{p_2 + t_2} \eta_2 \Leftrightarrow \tau_1 \eta_1 = \tau_2 \eta_2$$

- The optimal ad valorem/proportional tax rates (τ_i) solve:

$$\frac{\tau_1}{\tau_2} = \frac{\eta_2}{\eta_1} \text{ and } \tau_i = \frac{\lambda}{\eta_i}$$

Using the inverse elasticity/Ramsey rule?

- The inverse elasticity rule[4] is given by:

$$\frac{\tau_1}{\tau_2} = \frac{\eta_2}{\eta_1}$$

- The revenue constraint[5] is:

$$R = \tau_1 p_1 C_1^* + \tau_2 p_2 C_2^*$$

- Pick R then solve system of equations
- Tax elasticities of demand are sufficient statistics for optimal tax rates.

[4] Often called the Ramsey Rule

[5] ad valorem taxes are percentages of price

Application solve for tax rates

Suppose gov't needs to raise $R = \$250$ through taxes τ_1 and τ_2 on goods C_1 and C_2. Elasticities of demand are $\eta_1 = -2$ and $\eta_2 = -1$.

- What is the optimal ratio of ad valorem tax rates?

$$\frac{\tau_1}{\tau_2} = \frac{\eta_2}{\eta_1} = \frac{-1}{-2} = \frac{1}{2}$$

- $p_1 = \$10$ and $p_2 = \$20$ and $C_1 = C_2 = 100$. Solve τ_1, τ_2

$$R = \tau_1 \times 100 \times 10 + \tau_2 \times 100 \times 20 = \$250$$
$$\Rightarrow \left(\frac{1}{2}\tau_2\right) \times 100 \times 10 + \tau_2 \times 100 \times 20 = \$250$$
$$\Rightarrow \tau_2 = 0.1 \text{ and } \tau_1 = 0.05$$

- For larger taxes, should we be skeptical of the results? Yes! Assumptions break down for big taxes.

Complexity: income & substitution effects (no leisure)

- Assumed quasilinear and additively separable preferences to remove income and cross-price effects.
- **With income effects:** Observed elasticities aren't compensated (Hicksian) that isolate price effects.
- Cross-price effects require estimating taxes using the Slutsky matrix weighted by budget shares. (oof)
- **Heuristics:**
  - Small cross effects: $\tau_i \propto 1/|\eta_{ii}^c|$ (inverse-elasticity).
  - Strong cross effects: avoid large rate gaps on substitutes; adjust with η_{ij}^c.
- **Big picture:** Inverse elasticity rule's intuition holds, but math complicates without assumptions.

Complexity: supply elasticity

- Assumed perfectly elastic supply; only demand elasticity mattered.
- Supply elasticity also matters; more elastic supply should be taxed less.
- Again, just messier math

Complexity: salience

- Chetty, Looney, and Kroft (2009): tax presentation affects demand.
- Salience impacts optimal tax design; perceived tax affects effective elasticity and the ramsey rule:

$$\frac{\tau_1}{\tau_2} = \frac{\theta_2 \eta_2}{\theta_1 \eta_1}$$

.

- Tax incidence affects design and burden calculations.
- Without income effects, can tax low-salience goods more; with income effects, tax them less.
- Keep taxes low on low-salience goods.

Complexity: equity

- Inelastic goods (e.g., water, insulin) are essentials, larger budget share for low-income households.
- Should they be taxed more than elastic luxury goods (e.g., yachts)?
- Not necessarily; assumptions include a representative consumer and utilitarianism.
- Changing assumptions could adjust the inverse elasticity rule for social welfare costs.

Conceptual questions

- Should all goods be taxed at the same rate?
- How does the tax rate change with the elasticity of a good?
- Let's say you set a given revenue level R, how would you set up the problem?
- Say the market size for one good doubles ceteris paribus, do you cut the tax rate in that market in half?

Implications and Conclusion

- Uniform taxation and tax exemptions are difficult to justify.
- The inverse elasticity rule serves as a useful guideline in the absence of lump-sum taxes.
- Elasticity estimates are vital for effective tax policy.
- Equity concerns may require balancing taxes on different types of goods.
- Despite strong assumptions, the inverse elasticity rule offers valuable insights.