Social Insurance: Expected Utility and Insurance

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Bungee jumping and travel insurance

- Imagine a travel insurance company wants to provide people with coverage in the event they get hurt
- So they offer insurance contracts: clients pay a large sum of money and cover the cost of any major trip interrupting/ending events
- You happen to be on your honeymoon in New Zealand and drive by a bungee jumping bridge
- You have purchased said travel insurance?
- Do you jump?
- I did.


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Learning Goals

- Understand the role of insurance
- Define moral hazard and adverse selection problems
- Isolate reasons government is involved in different social insurance markets
- Identify potential moral hazard and adverse selection in social insurance programs/markets
- Characterize trade-offs in optimal insurance provision

Insurance and its jargon

What is insurance?

- Insurance is a promise to make some payment in case of a particular event, in exchange for a payment, called a premium.
- Insurance premiums: Money that is paid to an insurer so that an individual will be insured against adverse events.
- Insurance provides consumption smoothing
  - Auto insurance pays when a car is totaled (huge loss in income)
  - Health insurance pays when after expensive health costs (indirectly by paying for procedures)

Jargon

- **Payout**: the amount of money you receive after event
- **Premium**: the amount of money you pay for the insurance

Insurance in the Economy

Private-Sector Insurance: health care is 20% of the economy

- Health-insurance is a huge component of health care markets
- Non-health insurance: life insurance, auto insurance, home insurance, pet insurance, renters insurance, etc.

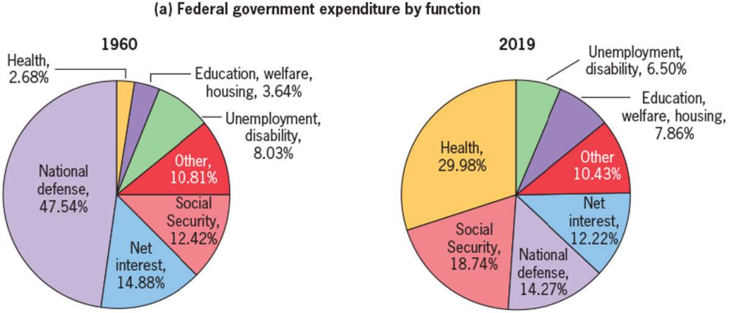
Government social insurance:

- participation compulsory, benefits not means-tested
- benefits depend on past contributions and begin with some identifiable event (e.g., unemployment, illness, retirement)

Ways governments intervene in insurance markets:

- Direct provision of insurance
- Mandatory participation
- Regulations + subsidies

Insurance is a Large Part of Government Spending



Federal government is "an insurance company with an army" (Source: Gruber, *Public Finance and Public Policy*)

- Social Security (retirement and disability)
- Medicare (and Medicaid, though means-tested), Veteran's Medical Care, ACA subsidies
- Unemployment Insurance, Worker's Compensation

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Decision Making under Uncertainty

- Thus far: decisions without uncertainty (should I consume coffee or donuts, how much do I work)
- Real life is uncertain: choosing jobs, retirement planning, location choices
- Insurance is about uncertainty: different payouts in different states of the world

Probability

Probabilities: the probability that an event occurs is p

- Interpretation: how likely the event is to occur
- Examples:
  - Probability a coin flip is heads = 50%
  - Probability a dice roll is 6 $= \frac{1}{6} \approx 16.66\%$

Properties of Probabilities

- Probability an event does not happen $= 1 - p$
- Probabilities need to sum to one

$$\sum_i p_i = p_1 + p_2 + \cdots + p_i = 1$$

- Sum over all possible events with event-specific probability p_i
- Example: with a coin flip: 50 % + 50 % = 100%

Lotteries

Lotteries: a lottery is a set of different monetary payments with a probability of each payment occurring

- 50% chance of \$1 million and 50% chance of \$0
- Each lottery is a list of payments $\{x_i\}$ and probabilities $\{p_i\}$
- Previous example:

$$\{x_i\} = \{\underbrace{1\text{M}}_{x_1}, \underbrace{0}_{x_2}\} \text{ and } \{p_i\} = \{\underbrace{0.5}_{p_1}, \underbrace{0.5}_{p_2}\}$$

- We can represent lots of uncertain choices as lotteries

Preferences over Lotteries: like consumption bundles, individuals can have preferences over lotteries

Expected Value

Expected Value: the mean of a random variable (lottery payoffs):

$$\underbrace{\mathbb{E}(X)}_{\textbf{Expected value of } X} = \sum_i \underbrace{x_i}_{\textbf{Payoff} x_i} \times \underbrace{p_i}_{\textbf{Prob. of payoff } x_i}$$

- Intuition: on average how much are do you get from a lottery?

Gaining .1 points w/ 75 % and losing .1 points w/ 25%:

$$.1 \times .75 - .1 \times .25 = .05$$

Gaining 20 points w/ 75 % and losing 20 points w/ 25%:

$$20 \times .75 - 20 \times .25 = 10$$

Expected utility theory and utility functions

Motivation: utility functions let us compare choices of consumption bundles

- It would be convenient to be able to assign a utility to a lottery

Expected Utility: the expected utility from a lottery L (any uncertain choice) is

$$\underbrace{\mathbb{E}[U(L)]}_{\textbf{Expected utility from } L} = \sum_i \underbrace{u(x_i)}_{\text{Utility from payoff } x_i} \times \underbrace{p_i}_{\text{Probability of payoff } x_i}$$

- Difference from expected value: expected *utility* not payoff

Prefer lottery L to L' if $\mathbb{E}[U(L)] > \mathbb{E}[U(L')]$

Question: Expected Utility

Consider an indivdual with a utility function $u(x) = \sqrt{w}$ and a lottery that pays \$16 with 50% chancce and \$0 with 50% chance. What is the expected utility and expected utility of this lottery?

Expected Utility:

$$\mathbb{E}[U(L)] = \sqrt{16} \times .5 + \sqrt{0} \times .5 = 2$$

Expected Value:

$$\mathbb{E}[L] = 16 \times .5 + 0 \times .5 = 8$$

Consumption Smoothing

Consumption Smoothing: Spend less in high income years and more in low income years:

- If \$75K/year is better than \$150K/year in one year and zero the next, then consumption smoothing is good

Risk Aversion: Diminishing marginal utility implies risk aversion

- Risk averse individuals like consumption smoothing: transferring money from low MU states to high MU states

Example

- This year, you earn \$100K.
  - Low state (A): Car stolen with probability p (lose \$36K)
  - High state (B): Car not stolen with probability $1 - p$
- On average, income[1] is $64p + 100(1 - p) = 100 - 36p$
- Utility is $u(C) = \sqrt{C}$, i.e. diminishing marginal utility
- Expected utility is less than utility of expected income

$$p\sqrt{64}+(1-p)\sqrt{100} \leq \sqrt{64p + 100(1 - p)} \quad \text{Check with } p = 0.5$$

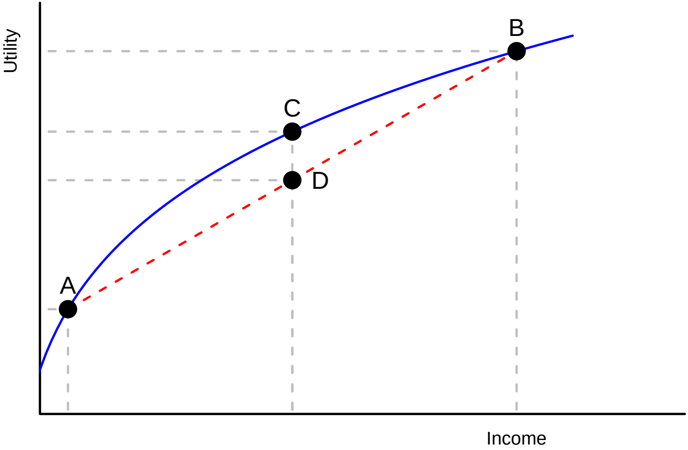
- What if you could spend \$18K to get \$36K if car is stolen:

$$\text{Car not stolen:} C_H = 100 - 18 = 82$$
$$\text{Car stolen:} C_L = 100 - 18 - 36 + 36 = 82$$

- Expected utility is now equal to utility of expected income!

[1] Units are thousands of dollars

Value of insurance with concave utility (diminishing MU)



Does this person prefer a lottery for A-B or C with certainty?

Example continued

- A (risk neutral) insurer can guarantee average income
  The contract: insured pays $36p$, receives 36 if robbed
- The expected cost to the insurer: $36p - 36p = 0$
- If $p = 0.5$, insurer can provide insurance at a premium of $18K
- The "actuarially fair" price!
- Intuition: insured pays premium equal to the average loss
- Insurer breaks even
- Insured trades off risk for consumption by shifting income from high to low state at a price

Would you play the following lotteries?

Lottery 1:

- You get an extra .1 point on your grade with 75% chance
- You lose .1 point on your grade with 25% chance

Lottery 2:

- You get an extra 20 points on your grade with 75% chance
- You lose 20 points on your grade with 25% chance

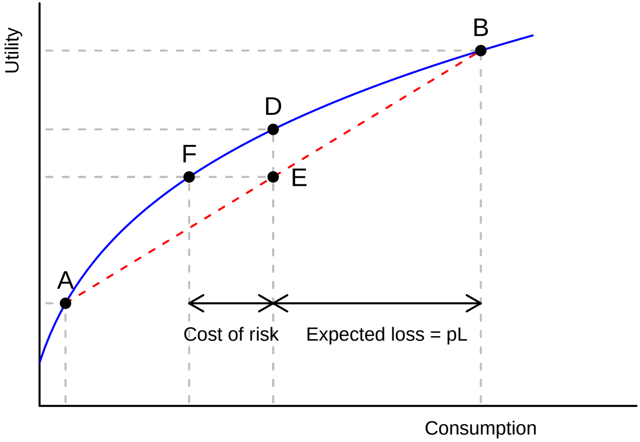
Large vs. Small Risks

- An individual has concave utility $U(C)$ and faces a potential loss L with probability p.
- What is the expected value of the loss? Expected loss is pL
- Define αL as the amount the person would pay to avoid the risk ("cost of risk"):

$$U(C - \alpha L) = pU(C - L) + (1 - p)U(C)$$

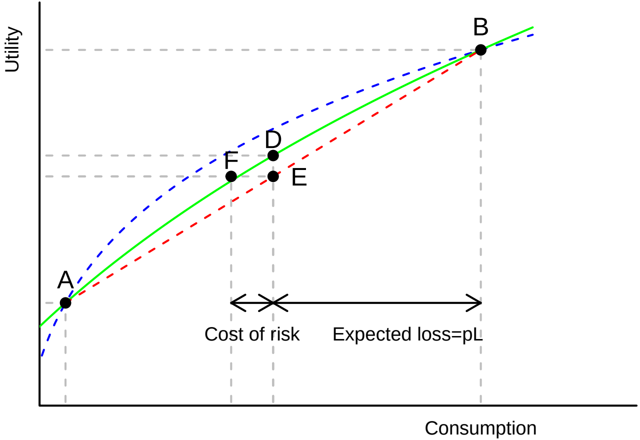
  $\Rightarrow C - \alpha L$ is the **certainty equivalent**.
- If $\alpha = p$, is the person risk averse? No!
- As $L \to 0$, what happens to α for a risk averse person? $\alpha \to p$
- **Large risks are worth insuring; small risks are not.**

Value of insurance



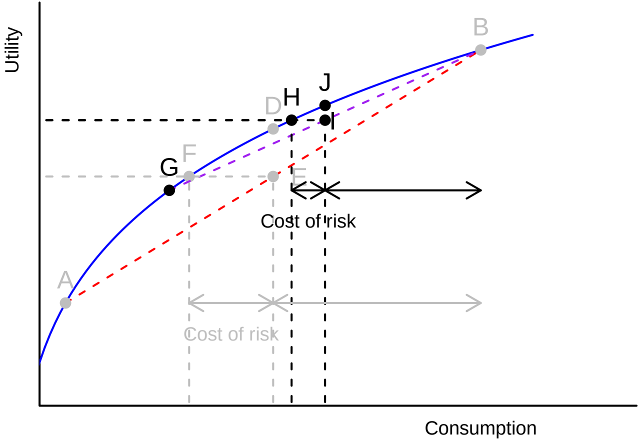
$$\underbrace{pU(C-L)+(1-p)U(C)}_{\mathbb{E}[U(C-L)]\text{ without insurance}} \leq \underbrace{U(C-pL)}_{\text{Utility of expected income}}$$

Value of insurance



As risk aversion falls, $\alpha \to p$ for any L, i.e. $F \to E$ on the graph

Value of insurance



For a smaller loss, the cost of risk, and thus certainty equivalent from $F \rightarrow E$ to $H \rightarrow I$, eventually converging such that H and J are the same point, so $\alpha = p$.

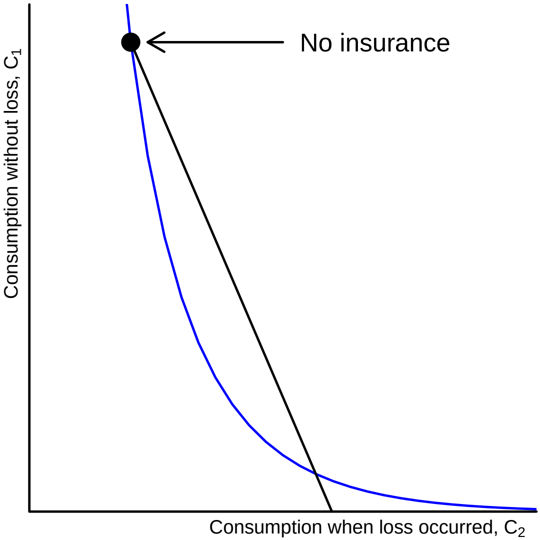
Generalize: Buying insurance

- Before insurance:
  - Consumption without a loss: $C_1 = Y$ (probability $1 - p$).
  - Consumption with a loss: $C_2 = Y - L$ (prob. p)
- Insurance with payout: R, premium: qR. If purchased:
  - $C_1 = Y - qR$
  - $C_2 = Y - L - qR + R = Y - L + (1 - q)R$
- If $R = 1$, what is the premium? q!
- Reducing C_1 by q increases C_2 by $(1 - q)$
- One can "trade C_1 for C_2" at the relative price of $\frac{1-q}{q}$
- Budget constraint: Pay $Y - C_1$ at price q to get $R = \frac{Y - C_1}{q}$:

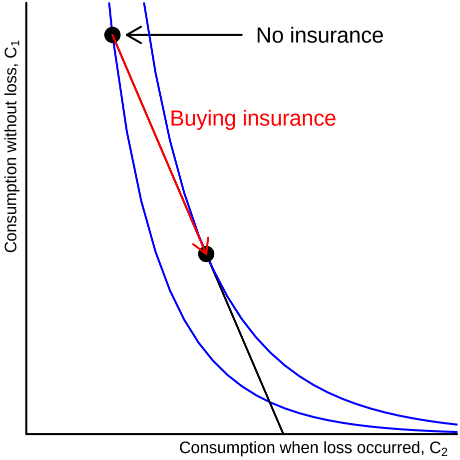
$$\begin{aligned} C_2 = Y - L - (1-q)R \quad &\Rightarrow C_2 = Y - L + \tfrac{1-q}{q}(Y - C_1) \\ \Rightarrow qC_2 + (1-q)C_1 \quad &= Y - qL \end{aligned}$$

- What does that look like? Budget constraint!

Buying insurance



Buying insurance



Wait, trade C_1 for C_2? Like an MRS?

- Recall the marginal rate of substitution!
- MRS is how much of one good you will give up for another
- Let's look at expected utility with just two states

$$\max_{C_1, C_2} (1-p) \cdot U(C_1) + p \cdot U(C_2)$$

- The MRS is the ratio of the marginal utilities or

$$-\frac{p}{1-p} \cdot \frac{MU_2}{MU_1}$$

- The MRS determines the rate at which you are willing to trade consumption in one state for another
- Huh... didn't paying q (out of C_1) get $1-q$ more of C_2?

$$-\frac{q}{1-q} = -\frac{p}{1-p} \cdot \frac{MU_2}{MU_1}$$

Buying insurance

- $Y = \$81\text{K}$, $L = \$45\text{K}$, probability of a loss is $p = 0.1$ and the price of insurance paying \$1 in case of a loss is $q = 0.2$.
  - If you buy insurance of R, $C_1 = 81 - 0.2R$ and $C_2 = 36 + 0.8R$.
  - The budget constraint is

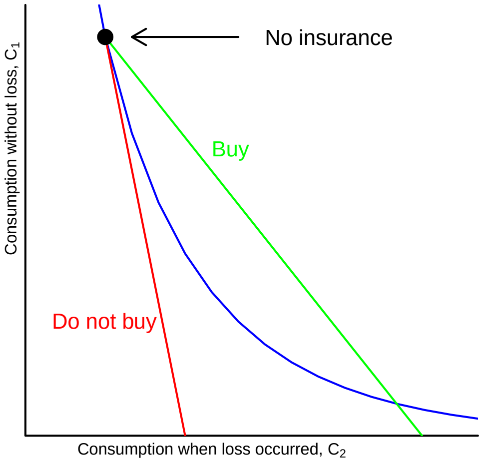
$$\begin{aligned} 0.8C_1 + 0.2C_2 &= 81 \cdot 0.8 + 36 \cdot 0.2 \\ &= 72 \end{aligned}$$

- $u(C) = \sqrt{C}$, so expected utility: $E[U] = 0.9\sqrt{C_1} + 0.1\sqrt{C_2}$
- Pick feasible C_1 and C_2 such that:

$$\underbrace{\overbrace{\frac{p}{1-p}}^{\text{Odds ratio}} \cdot \frac{\sqrt{C_2}}{\sqrt{C_1}} = \frac{.1}{.9}\frac{\sqrt{C_2}}{\sqrt{C_1}}}_{\text{MRS}} = \underbrace{\frac{q}{1-q} = \frac{.2}{.8}}_{\text{Slope of BC}}$$

- But what if C_1 and C_2 are not feasible?

Should you buy insurance?



Like utility maximization with two goods, we solve for the optimal C_1 and C_2 as the tangency between the indifference curve and budget constraint.

Solving the Insurance Problem

Assume $u(C) = \sqrt{C}$, $Y = \$81K$, $p = 0.1$, $q = 0.2$, $L = \$45K$.

Expected utility: $E[U] = 0.9\sqrt{C_1} + 0.1\sqrt{C_2}$

- Compare MRS to slope of budget line at $(C_1^*, C_2^*) = (81, 36)$:

$$MRS = -\frac{p}{1-p}\frac{MU_2}{MU_1} = -\frac{1}{9}\sqrt{\frac{C_1}{C_2}} = -\frac{1}{9}\sqrt{\frac{81}{36}} = -\frac{1}{6}$$

$$\text{Budget slope} = -\frac{q}{1-q} = -\frac{0.2}{0.8} = -\frac{1}{4}$$

- If $MRS >$ slope $\Rightarrow$ do not buy; if $MRS <$ slope $\Rightarrow$ buy!
- Does this person buy insurance? No! $-\frac{1}{6} > -\frac{1}{4}$
- What if $L = \$72K$? Yes! $-\frac{1}{3} < -\frac{1}{4}$
- Intuition: larger potential losses raise MU in bad state.

Solving for Optimal Coverage

$$\max_{C_1, C_2} \; (1-p)\sqrt{C_1} + p\sqrt{C_2}$$
$$\text{s.t. } (1-q)C_1 + qC_2 = Y - qL$$

Solve for C_1 and C_2, if $R = \frac{Y - C_1}{q} \leq 0$, no insurance!

Alternative: Substitute and max R

Substitute $C_1 = Y - qR$, $C_2 = Y - L + (1-q)R$:

$$\max_{R} \; 0.9\sqrt{Y - qR} + 0.1\sqrt{Y - L + (1-q)R}$$

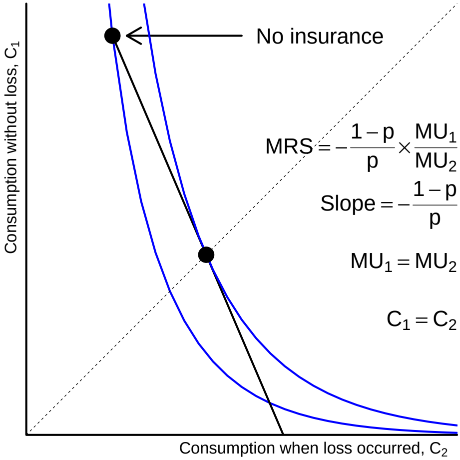
Actuarially fair insurance

- If insurers break even, price is "actuarially fair"
- to break even, we have to have

$$q \cdot R - p \cdot R = 0$$

  hence actuarially fair insurance requires $q = p$
- under actuarially fair insurance, consumers will insure fully and will equalize marginal utilities across states. Why?

Buying insurance



If actuarially fair, $p = q$, so the slope of the budget constraint is the odds ratio, $\frac{q}{1-q} = \frac{p}{1-p}$, so $MU_1 = MU_2$ meaning one fully insures.

Summary

- Insurance smooths consumption by transferring income from bad states (high MU) to good states (low MU).
- Solving for optimal insurance is the same as utility maximization with two goods – *you just need to properly define the budget constraint*
- With actuarially fair insurance, risk-averse consumers fully insure.
- Most insurance is not actuarially fair, but likely worth buying some coverage
- Insurance is more valuable for more risk-averse individuals and larger risks (e.g., smaller vs. larger changes in course points).
- Question: which aspects (if any) justify government intervention in insurance markets?